

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. SECOND SEMESTER EXAMINATION, MAY 2012

FIRST YEAR

MATHEMATICS (General)

Date : 26/05/2012

Time : 10.30 am – 1.30 pm

Paper : II

Full Marks : 75

[Use Separate Answer Books for each group]

Group – A

UNIT-I

(Answer **any three** questions)

1. a) The equation $3x^2 + 2xy + 3y^2 - 18x - 22y + 50 = 0$ is transferred to $4x^2 + 2y^2 = 1$ when referred to rectangular axes through the point (2, 3). Find the inclination of latter axes to the former. [4]
b) If the expression $ax + by$ changes to $a'x' + b'y'$ with rotation, prove that $a^2 + b^2$ is an invariant. [1]
2. Reduce the equation $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ to canonical form and determine the nature of the conic. [4+1]
3. a) Prove that the pair of straight lines joining the origin to the points of intersection of the parabola $y^2 = 4ax$ by the straight line $y = mx + c$ are at right angles if $c + 4am = 0$. [4]
b) Find the nature of the conic $\frac{17}{r} = \sqrt{5} - 2\cos\theta$. [1]
4. a) Find the centre of the circle $r = 3\sin\theta + 4\cos\theta$. [1]
b) Show that the sum of the reciprocals of two perpendicular focal chords of a conic is constant. [4]
5. Show that the pole of any tangent of the hyperbola $xy = c^2$ w.r.t the circle $x^2 + y^2 = a^2$ lies on concentric and similar hyperbola. [5]

UNIT-II

(Answer **any two** questions)

6. a) Solve the differential equation $(x + y)^2 \frac{dy}{dx} = a^2$. [3]
b) When the differential equation $Mdx + Ndy = 0$ is said to be exact. Investigate the exactness of the diff. equation $(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2)dy = 0$. [2]
7. Find the integrating factor of the differential equation $\cos^2 x \frac{dy}{dx} + y = \tan x$ and hence solve it. [2+3]
8. Find the general and singular solution of the differential equation $x^2(y - px) = p^2y$, $p = \frac{dy}{dx}$. [5]

Group - B

UNIT-I

(Answer **any three** questions)

9. a) For what value of c (scalar), the length of the vector $\vec{a} = c(2\hat{i} - 6\hat{j} + 3\hat{k})$ is of 5 unit? [2]
b) Find the moment about the point B(3, -1, 3) of a force P(4, 2, 1) passing through the point A(5, 2, 4). [3]
10. Prove that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar iff $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are coplanar. [5]

11. Show that by vector method the medians of a triangle are concurrent. [5]
12. a) Find the equation of the Plane passing through the point $(3, -2, -1)$ and Parallel to the vectors $(1, -2, 4)$ and $(3, 2, -5)$. [2]
- b) Find the shortest distance between two lines through $A(6, 2, 2)$ and $C(-4, 0, -1)$ and parallel to the vectors $(1, -2, 2)$ and $(3, -2, 2)$ respectively. [3]
13. If $\vec{a}, \vec{b}, \vec{c}$ be three non-coplaner vectors then show that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}\vec{b}\vec{c}]^2$. [5]

UNIT-II

(Answer **any five** questions)

14. Prove that every convergent sequence is bounded, but the converse is not true. [3+2]
15. a) Determine a, b so that $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$. [3+2]
- b) Find the maximum value of $x^{1/x}$. [3+2]
16. Using Lagrange's method of undetermined multipliers find the stationary point of $V = x^2 + y^2 + z^2$ subject to the condition $x + y + z = 6$. [4+1]
- Also determine whether V is maximum or minimum at this point. [4]
17. Expand e^x in Maclaurin's infinite series with proper justification. [2]
18. a) State Leibnitz's theorem on alternating series. [3]
- b) Test the convergence of the series $1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + \dots$. [5]
19. State and prove Lagrange's Mean Value theorem of a real valued function. [5]
20. Find the rectilinear asymptotes of $(x + y)^2(x + 2y + 2) = x + 9y + 2$ [5]
21. Find the envelope of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$
- Where a, b are variable parameters, connected by the relation $a + b = c$, c being a non-zero constant. [5]

UNIT-III

(Answer **any two** questions)

22. Evaluate the integral: [5]
- $$\int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx$$
23. Find the reduction formula for $\int \tan^n x dx$ (n is a positive integer greater than 1) and deduce the value of $\int_0^{\pi/4} \tan^6 x dx$. [3+2]
24. Using the method of integration evaluate $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right]$. [5]